

This coming quiz will require you to state the definition of the definite integral.

Then, the next quiz will require you to answer questions about the meaning of that definition.
(You will not be given the definition of the definite integral when you answer those questions.)

Definition of definite integral

If f is defined on $[a, b]$, then the definite integral of f over $[a, b]$ is

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $a + (i-1)\Delta x \leq x_i^* \leq a + i\Delta x$

if the limit exists and is the same regardless of how the x_i^* are chosen.

You should be able to answer each of the following questions about the definition in terms of finding the area under a non-negative function:

- [1] What does the index of the limit (n) represent ?
Why does the index of the limit approach the value that it does (∞) ?
- [2] Why is there a summation in the definition ?
- [3] What does the formula inside the summation ($f(x_i^*) \Delta x$) represent ?
- [4] What is the difference between using $f(x_i^*)$ and $f(a + i\Delta x)$ in the definition ?
What must you know to be true about f before you can use $f(a + i\Delta x)$ in the definition ?
- [5] When using $f(a + i\Delta x)$, what do the 2 different sets of lower/upper limits for the index of summation represent ? ie. $\sum_{i=1}^n f(a + i\Delta x) \Delta x$ versus $\sum_{i=0}^{n-1} f(a + i\Delta x) \Delta x$
- [6] What does the formula $\Delta x = \frac{b-a}{n}$ represent ?
- [7] What does the inequality regarding x_i^* ($a + (i-1)\Delta x \leq x_i^* \leq a + i\Delta x$) mean ?
- [8] Why must the formula for $\Delta x = \frac{b-a}{n}$ be written before the inequality regarding x_i^* ?
- [9] Why are the conditions at the end of the definition required ?
(In other words, if those conditions were not met, why would that present a problem for our definition ?)

I strongly encourage you to work together to answer these questions,
then bring your answers to me, and I will let you know if you need to fine tune anything.